

Bonds and the Price of Debt

A bond is the simplest security there is: a promise to pay stated amounts on stated dates. Because the promise is fixed, everything interesting about a bond happens to its price, and watching that price respond to changes in the world is the clearest available lesson in how any asset is valued.

The chapter has a second purpose, and in the design of this course it is the more important one. Chapter 6 argued that a discount rate is built from time, inflation, and a premium for risk. In the bond market that premium is not a theoretical construct requiring a model to estimate. **It is printed.** When a corporate bond yields three and a half percentage points more than a government bond of the same maturity in the same currency, the market has stated the price of that company's default risk, and you can read it off a screen.

HOW TO USE THIS CHAPTER

This chapter defines the instrument and its vocabulary, sets out the several different yield measures and what each is good for, develops duration and convexity as the general measures of interest-rate sensitivity, and shows how a credit spread converts into an implied probability of default.

The videos price a single government bond at three different yields, watch a holder take a loss without anything going wrong, and then compare a sovereign against a corporate of identical terms.

The case puts a distressed exchange offer in front of a bondholder who has to decide whether to accept it.

7.1 What a Bond Is

A bond is a loan divided into tradeable units. The borrower issues it, receives cash, pays interest on a schedule, and repays the face amount at maturity. What distinguishes it from a bank loan is that it can be sold, which means it has a market price, which means the market continuously states an opinion about it.

THE VOCABULARY

TERM	MEANING	NOTE
Face or par value	The amount repaid at maturity	Conventionally quoted per 100
Coupon rate	The annual interest, as a percentage of face	Fixed for the life of the bond
Maturity	The date the face amount is repaid	Determines sensitivity, see 7.5
Price	What the bond trades at today	Moves; the coupon does not
Yield to maturity	The return earned if held to maturity	The market's required rate
Indenture	The contract, including covenants	Where the protections live
Seniority	Position in the queue on default	Senior secured is repaid first

Several structural variants recur and are worth recognising, because each changes the risk in a specific way.

- **Zero-coupon.** No interest payments; issued at a discount and repaid at face. All of the return arrives at maturity, which makes it the most rate-sensitive structure available.
- **Amortising.** Principal is repaid in instalments alongside the coupon, so the outstanding balance falls over time. Common in project and infrastructure financing.
- **Callable.** The issuer may repay early, usually after a stated date. This is valuable to the issuer and therefore costs the buyer, who receives a higher yield in exchange for the risk of having a good investment taken away exactly when rates fall.
- **Floating-rate.** The coupon resets against a reference rate. The price is far more stable, because the instrument reprices itself rather than requiring its price to move.
- **Inflation-linked.** Principal or coupon is indexed to a price index. These instruments are how the real rate from Chapter 6 becomes directly observable.

7.2 Pricing a Bond

A bond is an annuity of coupons plus a single repayment of face value, so Chapter 6 has already supplied both formulas. Adding them gives the price.

$$\text{Price} = \text{Coupon} \times [1 - (1+y)^{-n}] / y + \text{Face} / (1+y)^n$$

Everything on the right is fixed by the terms of the bond except y , the yield the market currently demands. **The price is therefore entirely a statement about the required rate**, which is why bond prices are so often quoted in yield rather than in currency: the yield is the interesting number and the price is a translation of it.

Take a seven-year bond with a nine percent coupon and face value 100.

ONE BOND, FOUR REQUIRED YIELDS

MARKET REQUIRES	PRICE	TRADES AT
7%	110.78	A premium
9%	100.00	Par
11%	90.58	A discount
13%	82.31	A deeper discount

The middle row is the definition of par: when the required yield equals the coupon rate, the bond is worth exactly its face value. The two mechanisms on either side are worth stating, because they are the whole of bond arithmetic.

At a premium, the bond pays nine when comparable instruments pay seven. That surplus of two a year for seven years is worth about 10.78 today, so buyers bid the price up until the return on the price paid is seven percent — no more and no less. **At a discount**, the coupon is stingy relative to what is available, so nobody pays face value. The price falls until the buyer's total return — the nine

a year plus the gain from paying 90.58 and being repaid 100 — works out to the eleven percent the market demands.

Two relationships follow immediately and hold universally.

Price and yield always move in opposite directions.

And the price of a fixed-rate bond converges to face value as maturity approaches, regardless of what the yield does in the meantime, because the remaining cash flows shrink to a single repayment of a known amount on a near date.

IN THE VIDEOS Video 7.2 prices a government bond as an annuity plus a lump sum, and video 7.3 moves the required yield and watches the price move against it.

7.3 The Several Yields

"Yield" names more than one quantity, and the differences matter.

YIELD MEASURES

MEASURE	DEFINITION	USE AND LIMITATION
Coupon rate	Coupon / Face value	Fixed at issue; says nothing about return
Current yield	Coupon / Price	Income only; ignores the gain or loss to maturity
Yield to maturity	The rate equating price to all future cash flows	The standard measure; assumes reinvestment at that rate
Yield to call	The same, computed to the first call date	The relevant figure for a callable bond trading above par
Running spread	Yield less the government yield of the same maturity	The compensation for credit risk

The gap between current yield and yield to maturity is where the error usually occurs. The bond above, trading at 90.58, has a current yield of 9.94 percent and a yield to maturity of 11 percent. The difference is the 9.42 of capital gain earned by holding a discounted bond to repayment, spread across seven years. **Quoting the current yield on a discounted bond understates the return, and on a premium bond overstates it.**

One assumption inside yield to maturity is worth knowing because it is routinely forgotten. The measure assumes every coupon is reinvested at the yield itself. If rates fall, coupons are reinvested at less than that, and the realised return over the full period is below the quoted yield. The assumption is harmless for short maturities and material for long ones.

7.4 Interest-Rate Risk: Duration and Convexity

A bondholder who buys at par and watches yields rise has lost money without anything having gone wrong. The issuer has missed no payment, the terms have not changed, and no decision was

taken. **This is market risk rather than credit risk**, and separating the two is the central discipline of fixed-income analysis.

Whether the loss is real depends on the holder. An investor who sells realises it. An investor who holds to maturity receives every promised payment and earns exactly the yield they originally accepted; the lower market price is a quotation for a sale they will not make. The loss is nonetheless genuine in one respect — the money is now committed at a rate below what the market offers — which is why the position is marked to market even when it will never be sold.

Duration

The general measure of sensitivity is **duration**: the weighted average time until the bond's cash flows are received, with each time weighted by the present value of the flow arriving then.

$$\text{Macaulay duration} = \Sigma [t \times PVt] \div \text{Price}$$

Its practical form divides by one plus the yield, and turns duration into a direct estimate of the price move.

$$\text{Modified duration} = \text{Macaulay duration} / (1 + y) \quad \text{Price change} \approx - \text{Modified duration} \times \Delta y$$

For the nine percent bond above, priced at par, duration behaves as follows.

DURATION AT DIFFERENT MATURITIES

9% coupon, priced at par

MATURITY	MACAULAY DURATION	MODIFIED DURATION	ESTIMATED MOVE PER +1PP
3 years	2.759	2.531	-2.53%
7 years	5.486	5.033	-5.03%
15 years	8.786	8.061	-8.06%
25 years	10.707	9.823	-9.82%

Three properties of duration are worth reading off this table. **Duration is always less than maturity for a coupon bond**, because some cash arrives before the end; the only instrument whose duration equals its maturity is a zero-coupon bond, which is why zeros are the most rate-sensitive structure available. **Duration rises with maturity but at a decreasing rate** — going from 3 to 7 years nearly doubles it, going from 15 to 25 adds under a quarter — because distant flows are so heavily discounted that adding more of them changes the average little. And a **higher coupon shortens duration**, because more of the value arrives early.

Why long bonds move more

The duration table is the formal answer, and the intuition behind it is worth having separately. A change in the required yield applies to every remaining year. A bond with three years left has three years of mispriced coupons to correct for; one with twenty-five years has twenty-five, and each correction is compounded by the extra discounting. Hence the observed pattern:

THE SAME TWO-POINT RISE, DIFFERENT MATURITIES

9% coupon, yield moving from 9% to 11%

MATURITY	PRICE BEFORE	PRICE AFTER	CHANGE
3 years	100.00	95.11	(4.89%)
7 years	100.00	90.58	(9.42%)
15 years	100.00	85.62	(14.38%)
25 years	100.00	83.16	(16.84%)

The practical consequence is a matching rule. **Hold short maturities when you may need the money or expect rates to rise; hold long maturities when the obligation you are funding is itself long.** A pension fund with liabilities thirty years out is taking risk by holding short bonds, not by holding long ones, because its exposure is to the gap between the two durations rather than to either in isolation.

Convexity

Duration is a straight-line approximation to a curved relationship, and the curvature works in the holder's favour. Take the seven-year bond and move the yield two points in each direction.

EQUAL MOVES, UNEQUAL EFFECTS

YIELD MOVE	PRICE	CHANGE
Down 2 points, to 7%	110.78	+10.78%
Down 0, at 9%	100.00	—
Up 2 points, to 11%	90.58	(9.42%)

The gain from a fall exceeds the loss from an identical rise. This asymmetry is called convexity, and it arises because as the yield falls each further reduction is applied to a larger price. It is a genuine structural advantage of owning bonds, small over one or two percentage points and significant over larger moves. It is also why duration alone understates the value of a bond in a volatile rate environment, and why long-dated bonds — which have the most convexity — are worth slightly more than their duration suggests.

IN THE VIDEOS Video 7.5 compares the same yield move across four maturities and derives the matching rule from the result.

7.5 Credit Risk and Spreads

Everything so far assumed the payments arrive. The **credit spread** is the additional yield demanded for the possibility that they do not, and it is the observable price of risk that Chapter 6 promised.

Compare two bonds identical in every respect — same currency, same maturity, same coupon, same payment dates — differing only in who is promising. If the sovereign yields nine percent and the corporate yields 12.5, the spread is 350 basis points, and that 350 is the market's charge for this

issuer's default risk. The corporate bond, on the seven-year nine percent terms above, prices at 84.28 rather than 100 as a direct consequence.

What a spread implies

A spread can be converted into a rough annual probability of default, which is a useful sanity check because the resulting number can be argued about in ordinary business terms.

$$\text{Implied annual default probability} \approx \text{Spread} \div (1 - \text{Recovery rate})$$

The logic is that the expected loss from default must equal the extra compensation being paid. If a lender recovers 35 percent of face value in a default, each default costs 65 percent, so the annual default probability implied by a spread is the spread divided by 0.65.

SPREADS AS IMPLIED DEFAULT PROBABILITIES

At an assumed recovery rate of 35 percent

SPREAD	IMPLIED ANNUAL DEFAULT PROBABILITY	READS AS
150bp	2.31%	Solid investment grade
250bp	3.85%	Adequate, watched
350bp	5.38%	Meaningful credit risk
500bp	7.69%	Substantially impaired
800bp	12.31%	Distressed

Treat these as orders of magnitude rather than measurements, for three reasons. The recovery assumption is a guess and moves the answer proportionally. Spreads compensate for illiquidity and for uncertainty about the default probability, not only for the probability itself, so the implied figure is an upper bound. And spreads move with market sentiment in ways that have little to do with any individual issuer.

Even so, the conversion earns its place. **It turns a market price into a business proposition that can be argued with.** A colleague who cannot say whether a 350-basis-point spread is generous can usually say whether a five percent annual chance of this company failing sounds high or low, and that is a far more productive conversation.

Ratings, and what they are for

Credit ratings are an opinion on default probability expressed on an ordinal scale, from the highest grades down through the investment-grade boundary and into speculative grades. Two things are worth knowing about them.

- **The investment-grade boundary is an institutional fact, not an economic one.** Many funds are mandated to hold only investment-grade paper, so a downgrade across that line forces selling regardless of price. Spreads therefore widen far more than the change in default probability warrants.
- **A rating is relative, not absolute.** It ranks issuers; it does not promise a default rate. In markets with few rated issuers — which includes most of the continent — the practical substitute is a

synthetic rating built from financial ratios, and Chapter 10 uses that approach when no market price for a company's debt exists.

IN THE VIDEOS Video 7.4 puts a sovereign and a corporate bond side by side and reads the spread between them as a price.

7.6 How Bonds Reach the Market

Prices and yields are easier to interpret with some sense of where they come from. The market has two halves, and in most African markets the second is much thinner than the first — which affects every number read off it.

THE TWO HALVES OF THE MARKET

MARKET	WHAT HAPPENS	WHO PARTICIPATES
Primary	New securities are issued, usually by auction	Banks, pension funds, insurers, primary dealers
Secondary	Existing securities change hands	The same institutions, plus anyone holding

Governments in the region typically issue across a maturity ladder, and the vocabulary is worth knowing because the instruments behave differently.

- **Treasury bills**, from 91 days to a year. Issued at a discount with no coupon; the return is the difference between the price and the face amount. These are the shortest and most liquid instrument in most markets, and their yields are what a domestic risk-free rate is usually read from.
- **Notes and bonds**, from two years to twenty or more. Coupon-bearing, and the source of the yield curve when enough maturities trade.
- **Corporate bonds and commercial paper**. Present in most markets, thin in nearly all of them. The spread over the sovereign is exactly what section 7.5 describes, when it can be observed at all.

Auctions are usually settled at multiple prices — each successful bidder pays what they bid — or at a single clearing price for all. The **bid-to-cover ratio**, the total amount bid divided by the amount offered, is reported afterwards and is a useful indicator of appetite: a ratio near 1.0 means the issue barely cleared, and yields on the next auction typically rise.

Two features of thin secondary markets matter for the analysis in this chapter and beyond.

- **Quoted yields may not be transactable**. A yield derived from a bond that last traded three weeks ago is an estimate. When a spread is being read as the price of credit risk, check that the instrument actually trades.
- **Liquidity is itself priced**. Part of the yield on a rarely traded bond compensates for the difficulty of selling it, not for default risk. This is one reason section 7.5 described the implied default probability as an upper bound, and it is why the same issuer can appear to have a wider spread than its financial condition warrants.

The practical consequence for Chapter 10 is direct. A **domestic risk-free rate is only as good as the market that produced it**, and in markets where the sovereign itself carries real default risk, the local government yield is not a risk-free rate at all. That is precisely the problem Chapter 10 has to solve.

7.7 Why This Chapter Matters Later

- **A risk premium is observable.** Chapters 8 and 9 use discount rates containing a premium for risk. This chapter shows that for debt the premium can simply be looked up, which makes it far less mysterious when Chapter 10 estimates one for equity.
- **The cost of debt is a yield, not a coupon.** Chapter 10 requires the rate a firm would pay to borrow **today**, which is the yield on its outstanding bonds, not the coupon on debt it issued three years ago.
- **Discounting is one mechanism.** The bond in this chapter and the company in Chapter 9 are valued by exactly the same procedure. The bond is easier only because its cash flows are contractual.

7.8 Reference

CHAPTER 7 • FORMULA REFERENCE

QUANTITY	FORMULA	NOTE
Bond price	$C \times [1 - (1+y)^{-n}] / y + \text{Face} / (1+y)^n$	An annuity plus a lump sum
Current yield	Coupon / Price	Ignores gain or loss to maturity
Yield to maturity	The y that equates price to the cash flows	Assumes reinvestment at y
Macaulay duration	$\Sigma [t \times \text{PVt}] / \text{Price}$	Weighted average time to receipt
Modified duration	Macaulay / $(1 + y)$	Direct price sensitivity
Price change	$\approx - \text{Modified duration} \times \Delta y$	A straight-line approximation
Credit spread	Corporate yield – sovereign yield, same maturity	The observable price of default risk
Implied default probability	Spread / $(1 - \text{recovery rate})$	An upper bound; a sanity check

CARRY FORWARD

Price and yield move in opposite directions, always. A holder can lose money on a bond with no payment missed and no term changed.

Duration measures rate risk; the credit spread measures default risk. They are separate exposures and are managed separately.

A credit spread is a printed risk premium. Everything Chapter 10 does for equity, the bond market does for debt continuously and in public.

The cost of debt is today's yield, not yesterday's coupon. This distinction is worth a great deal in Chapter 10.